

Chapter 8

The celestial sphere: coordinate systems

8.1 Introduction

In the 4000 years during which astronomy has developed, various coordinate and timekeeping systems have been introduced because of the wide variety of problems to be solved. The coordinate systems have particular reference to great circles by which the direction of any celestial body can be defined uniquely at a given time. The choice of origin of the system also depends on the particular problem in hand. It may be the observer's position on the surface of the Earth (a **topocentric** system) or the Earth's centre (a **geocentric** system) or the Sun's centre (a **heliocentric** system) or, in the case of certain satellite problems, the centre of a planet (a **planetocentric** system). Indeed, in these modern days of manned space-flight, the origin can be a spacecraft (again a **topocentric** system) or centre of the Moon (a **selenocentric** system).

The time system used may be based on the movement of the Sun, on the Earth's rotation or on Dynamical Time (previously known as Ephemeris Time) which is related to the movements of the planets round the Sun and of the Moon about the Earth.

We now consider in some detail a number of coordinate and timekeeping systems.

8.2 The horizontal (alt-azimuth) system

This is the most primitive system, most immediately related to the observer's impression of being on a flat plane and at the centre of a vast hemisphere across which the heavenly bodies move.

In figure 8.1, the observer at O , northern latitude ϕ , can define the point directly opposite to the direction in which a plumb-line hangs as the **zenith**, Z . The plumb-line direction is known as the **nadir**, leading to the Earth's centre, if we assume the Earth to be spherical. On all sides, the plane stretches out to meet the base of the celestial hemisphere at the horizon.

Because of the Earth's rotation about its north-south axis PQ , the heavens appear to revolve in the opposite direction about a point P_1 which is the intersection of QP with the celestial sphere. Because the radius of the sphere is infinite compared with the radius of the Earth, this point is indistinguishable from a point P_2 , where OP_2 is parallel to QP . P_2 then said to be the **north celestial pole** and all stars trace out circles of various sizes centred on P_2 . Even *Polaris*, the Pole Star, is about one degree from the pole—a large angular distance astronomically speaking when we remember that four full moons (the angular diameter of the Moon is $\sim 30'$) could be laid side-by-side within *Polaris*' circular path about the north celestial pole.

The points N and S are the points where the great circle from the zenith through the north celestial pole meets the horizon, the north point, N , being the nearer of the two to the pole.

Since OP_2 is parallel to CP_1 ,

$$\angle ZOP_2 = \angle OCP_1 = 90 - \phi.$$

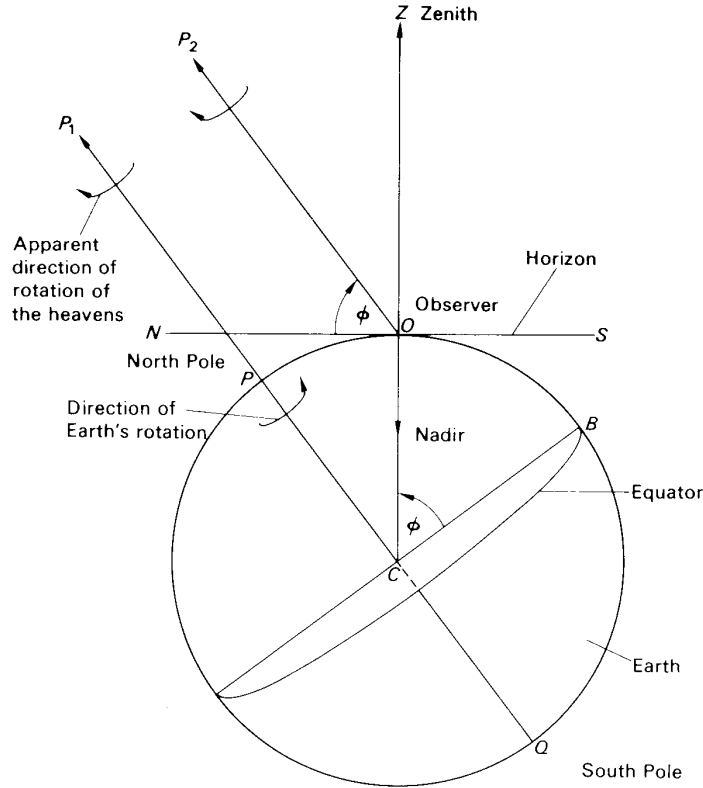


Figure 8.1. Definitions related to the observer’s position on the Earth.

Hence, $\angle NOP_2 = \phi$, since $\angle ZON$ is a right angle. That is,

The altitude of the pole is the latitude of the observer.

The horizontal system of coordinates has the observer at its origin so that it is a strictly local or topocentric system.

The observer’s celestial sphere is shown in figure 8.2 where Z is the zenith, O the observer, P is the north celestial pole and OX the instantaneous direction of a heavenly body. The great circle through Z and P cuts the horizon $NESAW$ at the north (N) and south (S) points. Another great circle WZE at right angles to the great circle $NPZS$ cuts the horizon in the west (W) and east (E) points. The arcs ZN, ZW, ZA , etc, are called **verticals**. The points N, E, S and W are the **cardinal points**. It is to be noted that west is always on the left hand of an observer facing north. The verticals through east and west are called **prime verticals**; ZE is the **prime vertical east**, ZW is the **prime vertical west**.

The two numbers that specify the position of X in this system are the **azimuth**, A , and the **altitude**, a . Azimuth is defined in a number of ways and care must be taken to find out which convention is followed in any particular use of this system.

For example, the azimuth may be defined as the angle between the vertical through the south point and the vertical through the object X , measured westwards along the horizon from 0° to 360° , or the angle between the vertical through the north point and the vertical through the object X , measured

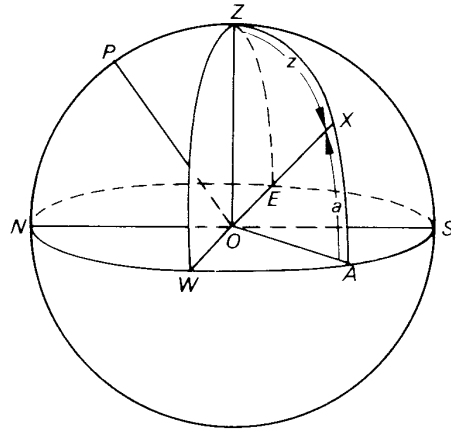


Figure 8.2. The observer's celestial sphere.

eastwards or westwards from 0° to 180° along the horizon. **A third definition commonly used is to measure the azimuth from the north point eastwards from 0° to 360° .** This definition will be kept in this text and is, in fact, similar to the definition of **true bearing**. For an observer in the southern hemisphere, the **azimuth is measured from the south point eastwards from 0° to 360° .**

The **altitude**, a , of X is the angle measured along the vertical circle through X from the horizon at A to X . It is measured in degrees. An alternative coordinate to altitude is the **zenith distance**, z , of X , indicated by ZX in figure 8.2. Obviously,

$$a = 90 - z.$$

The main disadvantage of the horizontal system of coordinates is that it is purely local. Two observers at different points on the Earth's surface will measure different altitudes and azimuths for the same star at the same time. In addition, an observer will find the star's coordinates changing with time as the celestial sphere appears to rotate. Even today, however, many observations are made in the **alt-azimuth** system as it is often called, ranging from those carried out using kinetheodolites to those made by the 250 feet (76 metre) radio telescope at Jodrell Bank, England. In the latter case, a special computer is employed to transform coordinates in this system to equatorial coordinates and *vice versa*. A similar solution is also employed with the large 6 metre (236 in) optical telescope at the Zelenchukskaya Astrophysical Observatory in the Caucasus Mountains. The William Herschel Telescope (4.2 m) on La Palma (Canary Islands) is also an alt-azimuth arrangement.

8.3 The equatorial system

If we extend the plane of the Earth's equator, it will cut the celestial sphere in a great circle called the **celestial equator**. This circle intersects the horizon circle in two points W and E (figure 8.3). It is easy to show that W and E are the west and east points. Points P and Z are the poles of the celestial equator and the horizon respectively. But W lies on both these great circles so that W is 90° from the points P and Z . Hence, W is a pole on the great circle ZPN and must, therefore, be 90° from all points on it—in particular from N and S . Hence, it is the west point. By a similar argument E is the east point.

Any great semicircle through P and Q is called a **meridian**. The meridian through the celestial object X is the great semicircle $PXBQ$ cutting the celestial equator in B (see figure 8.3).

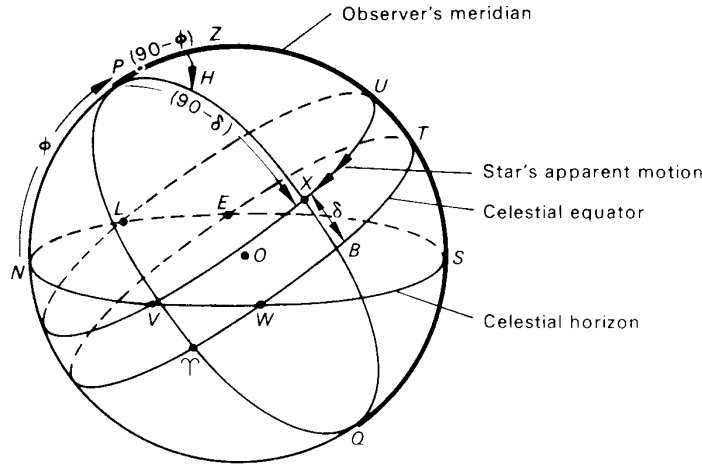


Figure 8.3. The equatorial system.

In particular, the meridian $PZTSQ$, indicated because of its importance by a heavier line, is the **observer's meridian**.

An observer viewing the sky will note that all natural objects rise in the east, climbing in altitude until they **transit** across the observer's meridian then decrease in altitude until they set in the west. A star, in fact, will follow a small circle parallel to the celestial equator in the arrow's direction. Such a circle (UXV in the diagram) is called a **parallel of declination** and provides us with one of the two coordinates in the equatorial system.

The **declination**, δ , of the star is the angular distance in degrees of the star from the equator along the meridian through the star. It is measured north and south of the equator from 0° to 90° , being taken to be positive when north. The declination of the celestial object is thus analogous to the latitude of a place on the Earth's surface, and indeed the latitude of any point on the surface of the Earth when a star is in its zenith is equal to the star's declination.

A quantity called the **north polar distance** of the object (X in figure 8.3) is often used. It is the arc PX . Obviously,

$$\text{north polar distance} = 90^\circ - \text{declination.}$$

It is to be noted that the north polar distance can exceed 90° .

The star, then, transits at U , sets at V , rises at L and transits again after one rotation of the Earth. The second coordinate recognizes this. The angle ZPX is called the **hour angle**, H , of the star and is measured from the observer's meridian westwards (for both north and south hemisphere observers) to the meridian through the star from 0^{h} to 24^{h} or from 0° to 360° . Consequently, the hour angle increases by 24^{h} each sidereal day for a star (see section 9.2).

8.4 Southern hemisphere celestial spheres

To clarify the ideas introduced in the previous sections, we consider the celestial sphere for an observer in the southern hemisphere. Let the latitude be ϕ S. Then the celestial pole above the horizon is the south celestial pole Q . We proceed as follows:

1. Draw a sphere.
2. Insert the zenith, Z , and the horizon, $NWSE$, a great circle with Z as one of its poles.

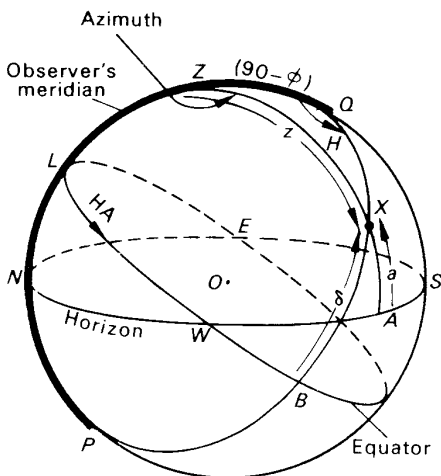


Figure 8.4. A southern hemisphere celestial sphere.

3. In figure 8.4 we have placed W on the front of the diagram. The convention that when facing north, west is on your left hand dictates the placing of N , S and E .
4. Insert Q , the *south* celestial pole, between the *south* point S and the zenith Z and such that the altitude SQ of the pole is the latitude ϕ of the observer. We can then insert the north celestial pole P directly opposite.
5. Put the celestial equator in the diagram, remembering that P and Q are its poles.
6. Insert the observer's meridian $QZNP$, according to the rule that it runs from the pole in the sky *through the zenith* and horizon to the pole below the Earth.
7. Put an arrowhead on the equator with HA beside it to show that the hour angle is measured *westwards* from the observer's meridian.
Let us suppose we are interested in the position of a star X .
8. Draw the vertical ZXA and the meridian $QXB P$ through its position. Then,

$$\begin{aligned} \text{the azimuth of } X &= \text{arc } SEA = A^\circ \text{ E of S} \\ \text{the altitude of } X &= \text{arc } AX = a^\circ \\ \text{the zenith distance of } X &= \text{arc } ZX = 90 - a \\ \text{the declination of } X &= \text{arc } BX = \delta^\circ \text{ S} \\ \text{the hour angle of } X &= \text{arc } LB = H^h \\ \text{the south polar distance of } X &= \text{arc } QX = 90 - \delta. \end{aligned}$$

Note: (a) that the azimuth is specified E of S to avoid ambiguity; (b) that the declination is labelled S again to avoid ambiguity.

8.5 Circumpolar stars

Consider the celestial sphere for an observer in latitude ϕ N (figure 8.5). The parallels of declination of a number of stars have been inserted with arrowheads to show that as time passes their hour angles increase steadily as the celestial sphere rotates.

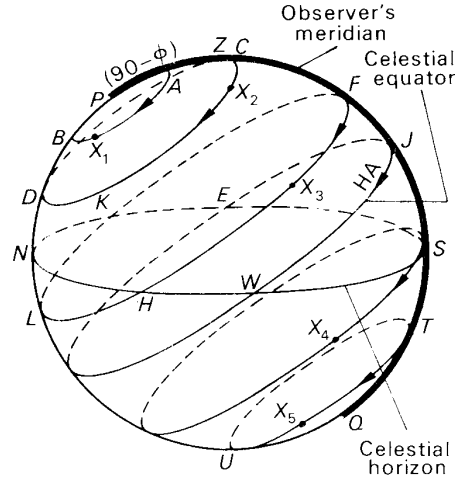


Figure 8.5. Circumpolar stars.

The stars can be put into three classes:

- stars that are above the horizon for all values of their hour angle,
- stars that are below the horizon for all values of their hour angle,
- stars that are seen to rise and set.

Stars in class (a) are **circumpolar** stars. Examples of these in figure 8.5 are stars X_1 and X_2 . Star X_1 transits at A north of the zenith in contrast to star X_2 's transit which is south of the zenith. These transits are referred to as **upper transit** or **upper culmination**. Both stars also transit below the pole; such transits are described as **below pole** or at **lower culmination**.

Now CD is the parallel of declination of star X_2 . In order that the star is circumpolar, then, we must have

$$PD < PN$$

that is

$$90 - \delta < \phi.$$

In order that the upper transit should be south of the zenith we must have

$$PC > PZ$$

that is

$$90 - \delta > 90 - \phi$$

or

$$\phi > \delta.$$

Stars in class (b) are never seen by the observer. The ancients who introduced the constellations were unaware of such stars, thus explaining why a roughly circular area of the celestial sphere in the vicinity of the south celestial pole is not represented in the ancient constellations.

In the diagram, star X_4 , of declination δ S, is the limiting case. Now

$$JS = 90 - \phi \quad SQ = 90 - \delta.$$

Also

$$JQ = 90^\circ.$$

Hence, we have

$$180 - \phi - \delta = 90$$

or

$$\phi + \delta = 90. \tag{8.1}$$

Hence, $\delta = 90 - \phi$ is the limiting declination of a star if it is to remain below the horizon.

If $\delta < 90 - \phi$, the star comes above the horizon. By putting a value in equation (8.1) for the declination of the stars at the edge of the roughly circular area not represented in the ancient constellations, it is found that the constellation-makers must have lived in a latitude somewhere between 34° and 36° N.

Most stars are found in class (c), that is they rise and transit, then set. For example, star X_3 moves along its parallel of declination (the small circle $FHLK$), setting at H , rising at K and transiting at F .

8.6 The measurement of latitude and declination

Let us suppose an observer in latitude ϕ N observes a circumpolar star of declination δ , for example star X_2 in figure 8.5. Using a *meridian circle* or *transit instrument* (section 20.5), the zenith distances ZC and ZD of the star at upper and lower culmination are measured. Now

$$PC = PZ + ZC$$

i.e.

$$90 - \delta = 90 - \phi + ZC.$$

Hence,

$$\phi - \delta = ZC. \tag{8.2}$$

Also,

$$PD = ZD - ZP$$

or

$$90 - \delta = ZD - 90 + \phi.$$

Hence,

$$\phi + \delta = 180 - ZD. \tag{8.3}$$

In equations (8.2) and (8.3) we have two equations in the two unknown quantities ϕ and δ . In principle, therefore, we can solve to obtain values of ϕ and δ .

In practice, a number of circumpolar stars are observed. Each star gives a pair of equations but only one extra unknown, namely the star's declination. Thus, with six stars, twelve equations in seven unknowns given by the latitude and the six declinations have to be solved. This is done by means of a mathematical procedure such as the method of least squares.

Example 8.1. In a place of latitude 48° N, a star of declination 60° N is observed. What is its zenith distance at upper and lower culmination?

In figure 8.6,

$$PZ = 90 - \phi = 42^\circ$$

$$PD = 90 - \delta = 30^\circ.$$

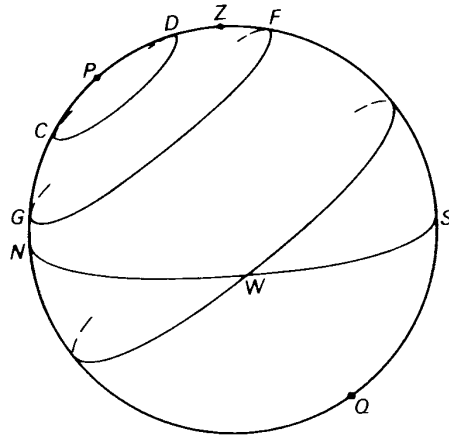


Figure 8.6. Examples 8.1 and 8.2—zenith distances at upper and lower culmination.

Hence,

$$DZ = 42^\circ - 30^\circ = 12^\circ.$$

The zenith distance at upper culmination is, therefore, 12° and is north of the zenith.

$$\begin{aligned} ZC &= ZP + PC = 90 - \phi + 90 - \delta \\ &= 180 - 48 - 60 \\ &= 72^\circ. \end{aligned}$$

The zenith distance at lower culmination is, therefore, 72° .

Example 8.2. The zenith distances of a star at upper culmination (south of the zenith) and lower culmination are 24° and 74° respectively. Calculate the latitude of the observer and the declination of the star.

Let the latitude and declination be ϕ and δ degrees respectively. Then

$$PF = PZ + ZF$$

that is

$$90 - \delta = 90 - \phi + ZF$$

or

$$\phi - \delta = 24^\circ. \quad (8.4)$$

Also,

$$PG = ZG - PZ$$

giving

$$90 - \delta = ZG - 90 + \phi$$

or

$$\phi + \delta = 180 - 74^\circ = 106^\circ. \quad (8.5)$$

Adding equations (8.4) and (8.5) we obtain

$$2\phi = 130^\circ \quad \text{or} \quad \phi = 65^\circ \text{ N.}$$

Substituting 65° for ϕ in equation (8.4) gives $\delta = 41^\circ \text{ N.}$

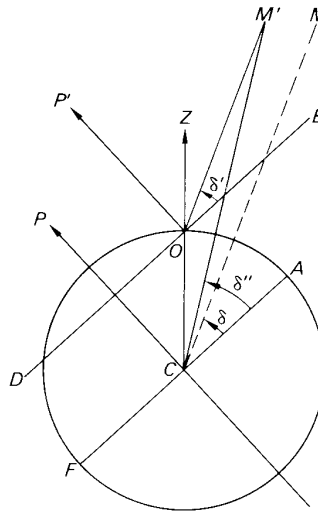


Figure 8.7. The geocentric celestial sphere showing the effect of parallax by moving the centre of a celestial sphere from an observer to the centre of the Earth.

8.7 The geocentric celestial sphere

So far we have assumed that the celestial sphere is centred at the observer. In the case of measurements made in the alt-azimuth system of coordinates, we have seen that as altitude and azimuth are linked to an observer's latitude and longitude, there are as many pairs of coordinates for a star's position at a given time as there are observers, even though the size of the Earth is vanishingly small compared with stellar distances.

Even in the equatorial system, the multitude of observing positions scattered over the Earth raises problems. For example, let us consider the declination of a star.

To the observer at O (see figure 8.7), the direction of the north celestial pole OP' is parallel to the direction CP in which it would be seen from the Earth's centre. Likewise the plane of the celestial equator DOB is parallel to the celestial equator obtained by extending in the Earth's equatorial plane FCA to cut the celestial sphere. A star's direction, as observed from O , would also be parallel to its direction as observed from C . Thus, OM' is parallel to CM where OM' is the direction to a particular star. So far, no problem is raised: the star's declination, δ' , as measured at O , is $\angle BOM'$, equal to the **geocentric** declination, δ , or $\angle ACM$.

There are a number of celestial objects, however, where a problem does arise. These objects are within the Solar System—for example, the Sun, the planets and their satellites, comets and meteors and, of course, artificial satellites and other spacecraft. None of these can be considered to be at an infinite distance. A shift in the observer's position from O to C , therefore, causes an apparent shift in their positions on the celestial sphere. We call such a movement (due, in fact, to a shift in the observer's position) a **parallactic** shift. Obviously, this apparent angular shift will be the greater, the closer the object is to the Earth. In particular, if M' is such an object, its declination, δ' , given by $\angle BOM'$ will no longer be equal to its geocentric declination, δ'' , given by $\angle ACM'$. In passing, it may be mentioned that $\angle OM'C$ is called the **parallactic angle**.

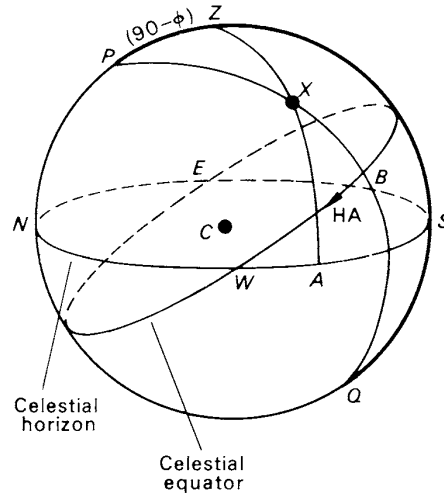


Figure 8.8. The geocentric celestial sphere and the position of a star.

In the almanacs such as *The Astronomical Almanac*¹, information about the positions of celestial objects, including the planets, is given for various dates throughout the year. It would be impossible to tabulate the declinations of a planet at a particular date for all possible observers. Hence, such information is tabulated for a hypothetical observer stationed at the Earth's centre. Various correction procedures are available so that any observer can convert the tabulated geocentric data to topocentric (or local) data.

From now on, unless otherwise stated, we will consider any celestial sphere to be a standard geocentric one. Thus, in figure 8.8 we have a geocentric celestial sphere for an observer in latitude ϕ N. The zenith is obtained by drawing a straight line from the Earth's centre through the observer on the Earth's surface to intersect the celestial sphere at Z. The celestial horizon *NWSE* is the great circle with Z as one of its poles, while the celestial equator is the intersection of the celestial sphere by the plane defined by the terrestrial equator. The observer's meridian is the great semicircle *PZSQ*.

For a star, X, then

- (i) its azimuth is the arc *NESA*,
- (ii) its altitude the arc *AX*,
- (iii) its zenith distance the arc *ZX*;
- (iv) its hour angle is $\angle ZPX$,
- (v) its declination the arc *BX*
- (vi) and its north polar distance the arc *PX*.

8.8 Transformation of one coordinate system into another

A common problem in spherical astronomy is a wish to obtain a star's coordinates in one system, given the coordinates in another system. The observer's latitude is usually known.

For example, we may wish to calculate the hour angle of *H* and declination δ of a body when its azimuth (east of north) and altitude are *A* and *a*. Assume the observer has a latitude ϕ N.

¹ Formerly the *Astronomical Ephemeris*, also published in the United States under the same title. Many similar almanacs are available in other countries.

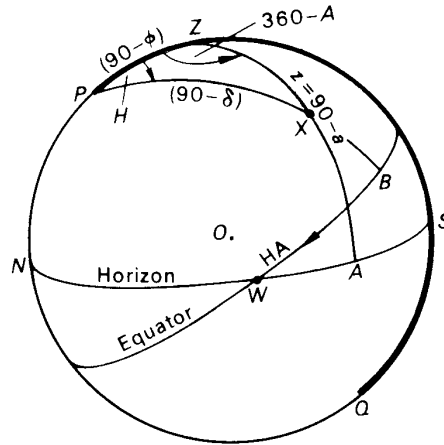


Figure 8.9. The conversion of azimuth and altitude to hour angle and declination.

The required celestial sphere is shown in figure 8.9 where X is the body's position.

In spherical triangle PZX , we see that we require to find arc PX and angle ZPX . We calculate PX first of all, using the cosine formula because we know two sides PZ , ZX and the included angle PZX .

Hence, we may write,

$$\cos PX = \cos PZ \cos ZX + \sin PZ \sin ZX \cos PZX$$

or

$$\sin \delta = \sin \phi \sin a + \cos \phi \cos a \cos A.$$

This equation enables δ to be calculated.

A second application of the cosine formula gives

$$\cos ZX = \cos PZ \cos PX + \sin PZ \sin PX \cos ZPX$$

or

$$\sin a = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H.$$

Re-arranging, we obtain

$$\cos H = \frac{\sin a - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

giving H , since δ is now known.

Alternatively, using the four-parts formula with ZX , $\angle PZX$, PZ and $\angle ZPX$, we obtain

$$\cos PZ \cos PZX = \sin PZ \cot ZX - \sin PZX \cot ZPX$$

or

$$\sin \phi \cos A = \cos \phi \tan a + \sin A \cot H$$

giving

$$\tan H = \frac{\sin A}{\sin \phi \cos A - \cos \phi \tan a}.$$

We consider the problem in reverse by means of a numerical example.

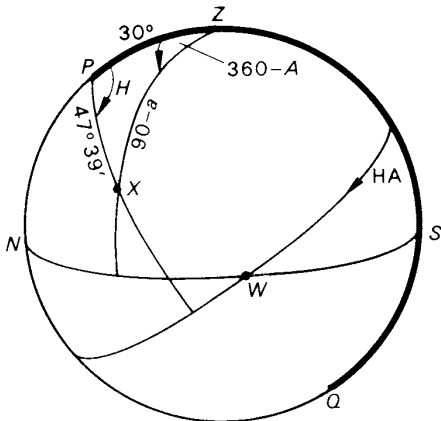


Figure 8.10. Example 8.3—conversion of hour angle and declination to azimuth and altitude.

Example 8.3. A star of declination $42^\circ 21' \text{ N}$ is observed when its hour angle is $8^{\text{h}} 16^{\text{m}} 42^{\text{s}}$. If the observer's latitude is 60° N , calculate the star's azimuth and altitude at the time of observation.

It is often a great help to sketch as accurately as possible a celestial sphere diagram of the problem. This provides a visual check on deductions about quadrants in which an angle lies.

Since $PX = 90 - \delta$, we see that its value is $47^\circ 39'$.

We convert the hour angle value of $8^{\text{h}} 16^{\text{m}} 42^{\text{s}}$ to angular measure by means of table 7.1 (see page 48).

$$\begin{aligned} 8^{\text{h}} 16^{\text{m}} 42^{\text{s}} &= 8^{\text{h}} + 16^{\text{m}} + 42^{\text{s}} \\ &= (8 \times 15)^\circ + (16/4)^\circ + (40/4)' + (2 \times 15)'' \\ &= 120^\circ + 4^\circ + 10' + 30'' \\ &= 124^\circ 10'5. \end{aligned}$$

Hence, $H = \angle ZPX = 124^\circ 10'5$, in figure 8.10 where X is the star.

$$PZ = 90^\circ - \phi = 90^\circ - 60^\circ = 30^\circ.$$

Applying the cosine formula to $\triangle PZX$, we may write

$$\cos ZX = \cos 30^\circ \cos 47^\circ 39' + \sin 30^\circ \sin 47^\circ 39' \cos 124^\circ 10'5$$

or

$$\sin a = \cos 30^\circ \cos 47^\circ 39' - \sin 30^\circ \sin 47^\circ 39' \cos 55^\circ 49'5.$$

The calculation then proceeds as follows:

$$\sin a = \cos 30^\circ \cos 47^\circ 6500 - \sin 30^\circ \sin 47^\circ 6500 \cos 55^\circ 8253$$

giving, on reduction,

$$a = 22^\circ 04'6.$$

Applying the cosine formula once more to $\triangle PZX$, we have

$$\cos 47^\circ 39' = \cos 30^\circ \cos(90 - a) + \sin 30^\circ \sin(90 - a) \cos(360 - A)$$

or

$$\cos 47^\circ 39' = \cos 30^\circ \sin 22^\circ 04'6 + \sin 30^\circ \cos 22^\circ 04'6 \cos A$$

giving

$$\cos A = \frac{\cos 47^\circ 65 - \cos 30^\circ \sin 22^\circ 0760}{\sin 30^\circ \cos 22^\circ 0760}.$$

On reduction we find that $A = 41^\circ 28'47$ or $360^\circ - 41^\circ 28'47$, that is $A = 41^\circ 17'1$ or $318^\circ 42'9$.

It is obvious from the diagram and the value of the hour angle that the correct value is $318^\circ 43'$ east of north to the nearest minute.

Check: Using the sine formula, we may write

$$\frac{\sin H}{\sin(90 - a)} = \frac{\sin(360 - A)}{\sin 47^\circ 39'}$$

that is

$$\sin H = \frac{-\cos a \sin A}{\sin 47^\circ 39'} = \frac{-\cos 22^\circ 0760 \sin 41^\circ 28'47}{\sin 47^\circ 65}$$

giving, on reduction, $H = 180^\circ - 55^\circ 8'248$ since $\sin H$ is negative. Hence, $H = 124^\circ 17'52 = 124^\circ 10'5$.

We see that this agrees with the original value of H .

8.9 Right ascension

We have seen that in the equatorial system, one of the coordinates of the star, namely the declination, is constant with time. The other, the hour angle, changes steadily with the passage of time and so is unsuitable for use in a catalogue of stellar positions.

The problem is solved in a manner analogous to the way in which places on the Earth's surface are defined uniquely in position, although the Earth is rotating on its axis. Latitude is defined with respect to the terrestrial equator. In spherical astronomy, declination, referred to the celestial equator, carries out the same task in fixing the place of a celestial object. The longitude of a place on the Earth's surface is defined with respect to a meridian through a particular geographical position, namely the Airy Transit Instrument at Greenwich, England, and the meridian through the place in question.

The Greenwich meridian cannot be used for celestial position-fixing. Because of the Earth's rotation under the celestial sphere, the projection of the Greenwich meridian sweeps round the sphere, passing through each star's position in turn. Some other meridian must be chosen which is connected directly to the celestial sphere.

If a point, Υ , fixed with respect to the stellar background, is chosen on the celestial equator, its angular distance from the intersection of a star's meridian and the equator will not change, in contrast to the changing hour angle of that star. In general, then, all celestial objects may have their positions on the celestial sphere specified by their declinations and by the angles between their meridians and the meridian through Υ . The point chosen is the **vernal equinox**, also referred to as the **First Point of Aries**, and the angle between it and the intersection of the meridian through a celestial object and the equator is called the **right ascension (RA)** of the object. Right ascension is measured from 0^h to 24^h or from 0° to 360° along the equator from Υ eastwards, i.e. in the direction opposite to that in which the hour angle is measured. Like the definition of hour angle, this convention holds for observers in both northern and southern hemispheres. In drawing a celestial sphere it is advisable not only to mark the observer's meridian heavily, inserting on the equator a westwards arrow with HA (hour angle) beside it but also to mark on the equator an eastwards arrow with RA (right ascension) beside it.

We now show that the choice of Υ as a reference point is closely connected with the Sun's yearly journey round the stellar background of the celestial sphere.

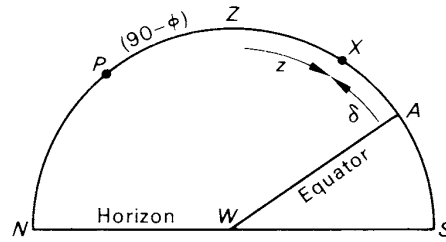


Figure 8.11. The Sun on the meridian.

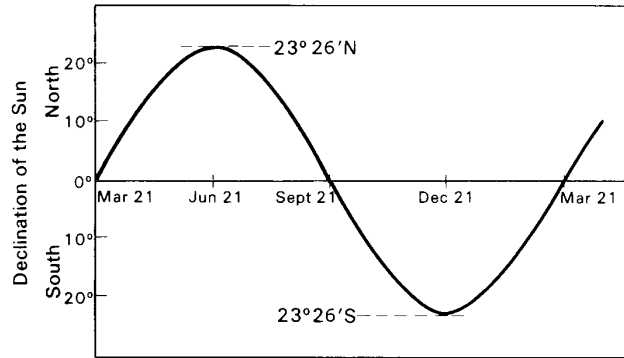


Figure 8.12. The variation of the Sun's declination through the year.

8.10 The Sun's geocentric behaviour

We have already seen that an observer studying the Sun and stars during a year comes to certain conclusions about the Sun's behaviour. The Sun appears to revolve about the Earth once per day in company with the stellar background but also has a slower motion with respect to the stellar background, tracing out a yearly path—the **ecliptic**—among the stars.

More precisely, let us suppose that an observer in a northern latitude (i) measures the meridian zenith distance of the Sun every day, i.e. at apparent noon, and (ii) notes at apparent midnight the constellations of stars that are on the meridian; and that he/she carries out this series of observations for one year. It should be noted that at **apparent noon**, the Sun's hour angle has a value zero; at **apparent midnight** its value is 12^{h} .

Then, from the midday measurement of zenith distance, the observer can keep track of the Sun's declination changes throughout the year. Assuming the latitude ϕ to be known, it can be seen (see figure 8.11), that if X is the Sun's position at transit on a particular day,

$$PA = 90^\circ = 90 - \phi + z + \delta$$

or

$$\delta = \phi - z$$

where δ and z are the Sun's declination and zenith distance respectively.

A graph of declination against date is obtained, illustrated in figure 8.12.

The record of those stars appearing on the observer's meridian at apparent midnight throughout the year shows that the Sun must make one complete revolution of the stellar background in that time.

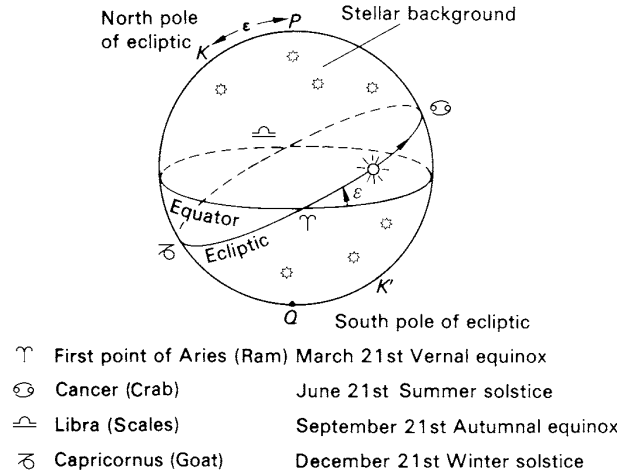


Figure 8.13. The solar path along the ecliptic.

Together with the declination record, it enables the Sun's yearly path on the stellar background to be traced out. This path, the ecliptic, is shown in figure 8.13.

The yearly journey takes the Sun through the twelve houses of the Zodiac, ancient constellations embodying the stars within a few degrees (8° – 9°) of the ecliptic. In order, they are given here, with symbols and meanings:

Aries	Taurus	Gemini	Cancer	Leo	Virgo
♈	♉	♊	♋	♌	♍
Ram	Bull	Twins	Crab	Lion	Virgin
Libra	Scorpius	Sagittarius	Capricornus	Aquarius	Pisces
♎	♏	♐	♑	♒	♓
Scales	Scorpion	Archer	Goat	Water-bearer	Fishes

The word 'Zodiac' means 'circle of the animals' and is the region of the celestial sphere in which the Sun, the Moon and those planets known to the ancients (Mercury, Venus, Mars, Jupiter and Saturn) are found.

Because of the effects of precession and as a result of the International Astronomical Union's (IAU) adoption of constellation boundaries (see chapter 6) which are not exactly 30° long, the Sun's passage through the constellations does not correspond to its passage through the signs of the Zodiac. According to the IAU boundary definitions, the Sun travels 44° through *Virgo*, only 7° through *Scorpius* and 18° through *Ophiuchus*, a constellation for which there is no Zodiacal sign.

It is, therefore, seen that for half the year the Sun is below the plane of the terrestrial equator and, consequently, that of the celestial equator. Its declination is south during this time, achieving its maximum southerly value about December 21st. This is the time of the **winter solstice** in the northern hemisphere (and the **summer solstice** for those people living in the southern hemisphere). The word 'solstice' means 'the standing still of the Sun' and refers to the pause in the Sun's progress in declination. Around March 21st the Sun crosses the equator, its declination changing from south to north. This is the **spring equinox** (**autumnal equinox** for southern hemisphere inhabitants). The word 'equinox' means 'equal day and night' because at the time of occurrence of the vernal and autumnal equinoxes, day and night are equal in length, everywhere on the Earth.

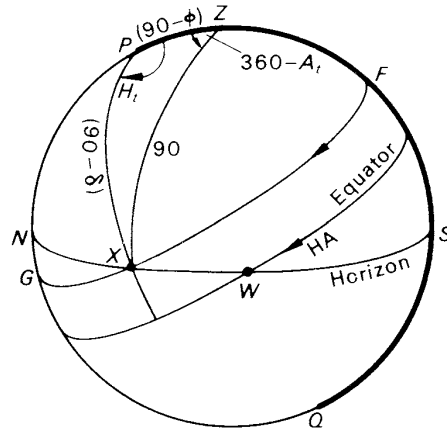


Figure 8.14. The diurnal motion of the Sun.

For six months after March 21st the Sun's declination is positive, that is northerly, becoming a maximum around June 21st, the date of the summer solstice in the northern hemisphere, thereafter decreasing to zero about September 21st, the autumnal equinox for the northern hemisphere.

The ecliptic, therefore, is a great circle defining a plane that intersects the plane of the celestial equator at an angle of about $23^{\circ} 26'$. This angle, usually denoted by ϵ , is called the **obliquity of the ecliptic**. The two points of intersection of the ecliptic with the equator are the First Point of Aries, Υ , and Libra, φ .

The zero point, from which right ascension is measured, is Υ , the point at which the Sun crosses the equator from south to north on its yearly journey round the ecliptic. Right ascension is, therefore, measured along the equator in the same direction in which the Sun travels round the ecliptic. In one year, consequently, the Sun's right ascension increases from 0^{h} (March 21st) through 6^{h} (June 21st), 12^{h} (September 21st), 18^{h} (December 21st) to 24^{h} (the succeeding March 21st).

8.11 Sunset and sunrise

Because of the changing declination of the Sun throughout the year, the hour angle and azimuth of sunset and sunrise will also change.

Strictly speaking, the Sun's declination is an ever-changing quantity, except at the solstices when it has stationary values but the changes are so slow that for many problems it is sufficiently accurate to consider the declination to be constant throughout the day. The Sun's diurnal path, to this degree of accuracy, is along a parallel of declination such as illustrated by the small circle FG in figure 8.14.

Let the Sun's declination be δ N on a particular day and let the Sun set on that day at X , so that $PX = 90^{\circ} - \delta$ and $ZX = 90^{\circ}$.

The Sun's hour angle at setting is $H_i = \angle ZPX$; its azimuth at setting is A_i given by $A_i = 360^{\circ} - \angle PZX$. Its zenith distance ZX , at setting is 90° .

Using the cosine formula in $\triangle PZX$, we have

$$\cos 90 = \cos(90 - \phi) \cos(90 - \delta) + \sin(90 - \phi) \sin(90 - \delta) \cos H_i$$

or

$$\cos H_i = -\tan \delta \tan \phi \quad (8.6)$$

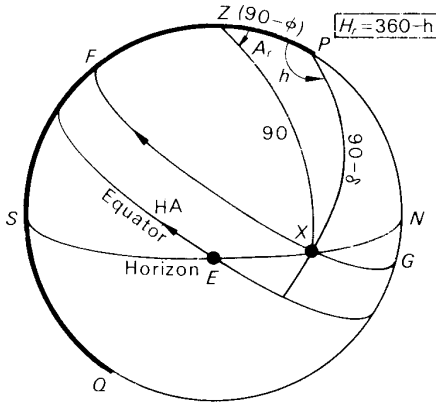


Figure 8.15. The celestial sphere illustrating sunrise.

showing that in a given latitude, the value of the Sun's hour angle at setting depends on the Sun's declination.

Thus, if the latitude is north and the declination is positive,

$$6^{\text{h}} \leq H_t \leq 12^{\text{h}}.$$

If the latitude is north and the declination is negative,

$$0^{\text{h}} \leq H_t \leq 6^{\text{h}}.$$

To obtain the azimuth at sunset, we apply the cosine formula to $\triangle PZX$ again. Thus,

$$\cos(90 - \delta) = \cos(90 - \phi) \cos 90 + \sin(90 - \phi) \sin 90 \cos(360 - A_t)$$

or

$$\cos A_t = \frac{\sin \delta}{\cos \phi}.$$

Note that for sunset, the azimuth, measured eastwards from north, is necessarily greater than 180° . For northern latitudes, if the declination is positive,

$$270^\circ \leq A_t \leq 360^\circ$$

but if the declination is negative,

$$180^\circ \leq A_t \leq 270^\circ.$$

The total number of hours of daylight in a given 24^h period is obtained approximately by doubling H_t , the hour angle of sunset (see equation (8.6)), for it is easily seen from the diagram for sunrise (figure 8.15) that the hours of daylight for sunrise to apparent noon equal H_t , being given by h or $\angle ZPX$.

The hour angle at sunrise, H_r , is given by

$$H_r = 360 - h$$

where

$$\cos h = -\tan \phi \tan \delta.$$

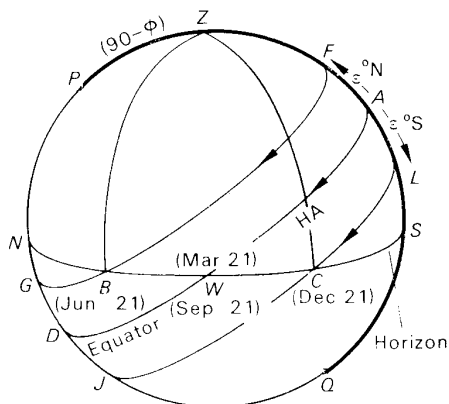


Figure 8.16. The azimuth limits of sunset.

The azimuth at sunrise, A_r , is given by

$$\cos A_r = \frac{\sin \delta}{\cos \phi}.$$

It is obvious that for northern latitudes, with positive declinations,

$$\begin{aligned} 0 \leq A_r \leq 90^\circ \\ 12^{\text{h}} \leq H_r \leq 18^{\text{h}} \end{aligned}$$

while for negative declinations,

$$\begin{aligned} 90^\circ \leq A_r \leq 180^\circ \\ 18^{\text{h}} \leq H_r \leq 24^{\text{h}}. \end{aligned}$$

Thus, in figure 8.16 for a place in northern latitude ϕ , the sunset point on the western horizon will move back and forth between B and C where FG and LJ are the most northerly and southerly parallels of declination of the Sun respectively, and $AF = AL = \varepsilon$, the obliquity of the ecliptic.

A corresponding cycle in the azimuth at the sunrise point takes place along the eastern horizon.

8.12 Megalithic man and the Sun

The Sun, as god and giver of light, warmth and harvest, was all-important. He was a Being to be worshipped and placated by ritual and sacrifice. On a more practical note, his movements provided a calendar by which ritual and seed time and harvest could be regulated. Throughout the United Kingdom, for example, between the middles of the third and second millennia BC, vast numbers of solar observatories were built. Some were simple, consisting of a single alignment of stones; others were much more complicated, involving multiple stone circles with outlying stones making ingenious use of natural foresites along the horizon to increase their observational accuracy. A surprising number of such megalithic sites still exist, the most famous of which are Stonehenge, in England, and Callanish, in the Outer Hebrides, Scotland.

One example will be sufficient to illustrate megalithic man's ingenuity. It exists at Ballochroy on the west coast of the Mull of Kintyre, Scotland.

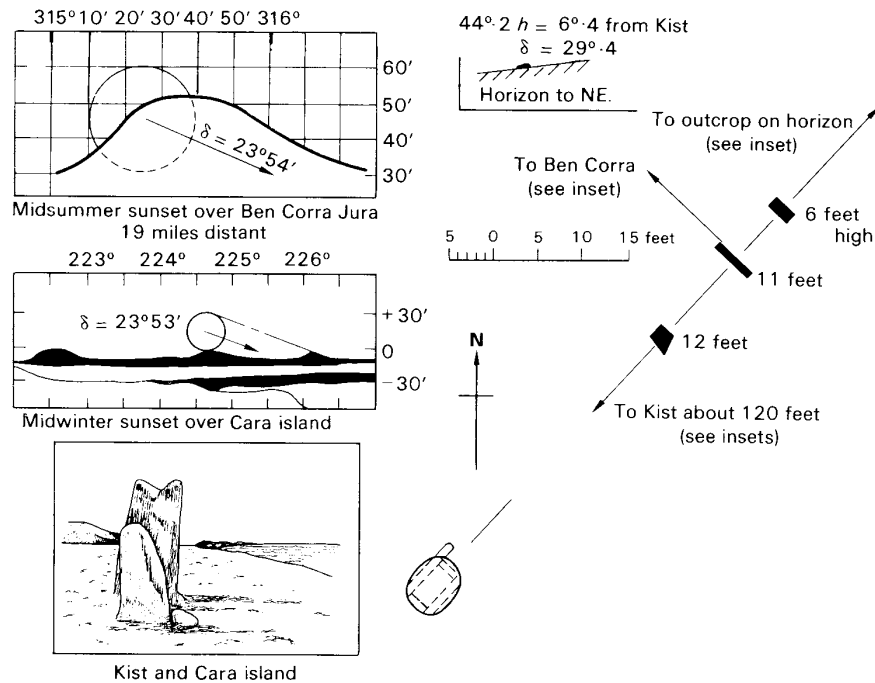


Figure 8.17. Megalithic man's sighting stones at Ballochroy ($55^{\circ} 42' 44''$ N, $5^{\circ} 36' 45''$ W).

Three large stones (see figure 8.17) are set up in line close together; a stone kist (a grave built of stone slabs) is found on this line to the south-west, about 40 m from the stones. The slabs are set parallel to each other. Looking along the flat face of the central stone we see the outline of Ben Corra in Jura, some 30 km away. Looking along the direction indicated by the line of stones and the kist, we see off-shore the island of Cara.

About 1800 BC, the obliquity of the ecliptic was slightly different in value from the value it has now. It had, in fact, a value near to $23^{\circ} 54'$. This would, therefore, be the maximum northerly and southerly declination of the Sun at that era on midsummer's and midwinter's day respectively (for the northern hemisphere). The corresponding values of the midsummer and midwinter sunset azimuths indicate that on midsummer's day, from a pre-determined position near the stones, the Sun would be seen to set behind Ben Corra in the manner indicated in figure 8.17 and such that momentarily a small part of its upper edge would reappear further down the slope. Midwinter's day would be known to have arrived when the megalithic observers saw the Sun set behind Cara Island in the manner shown.

We do not know the exact procedure adopted by the megalithic astronomers but it is likely to have been based on the following method. As the Sun sets on the western horizon, the observer, by moving along the line at right angles to the Sun's direction, could insert a stake at the position he/she had to occupy in order to see the upper edge or limb of the Sun reappear momentarily behind the mountain slope indicated by the stone alignment (figure 8.17). This procedure would be repeated for several evenings until midsummer's day. On each occasion the stake would have to be moved further left or a fresh one put in, but by midsummer's eve, a limit would be reached because the Sun would be setting at its maximum declination north. On the evenings thereafter, the positioning of the pegs would be retraced. In this way it would be known when midsummer's day occurred. It may be noted in passing that with a distant foresight such as the mountain 30 km or so away, even a shift of 12 arc sec in the

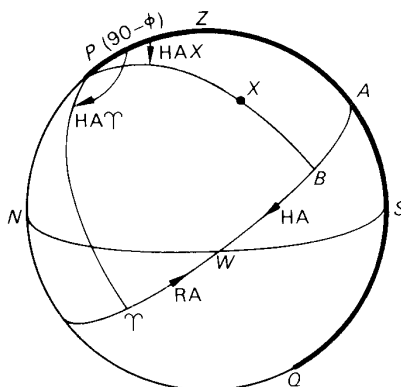


Figure 8.18. The celestial sphere illustrating local sidereal time.

Sun's setting the position will mean a stake-shift of the order of 2 m. This demonstrates the sensitivity of the method and the ingenuity of this ancient culture.

8.13 Sidereal time

Right ascension, together with declination, forms a coordinate system for stellar positions useful in constructing star catalogues, in contrast to the alt-azimuth and equatorial systems where one or both coordinates change rapidly with time.

The First Point of Aries, Υ , being a point on the stellar background, will rotate with the heavens like a star, transiting and rising and setting. We can, therefore, give a precise meaning to the phrase, 'the hour angle of Υ ($HA\Upsilon$)'. It is the angle which the meridian through Υ makes with the observer's meridian, $\angle ZP\Upsilon$ in figure 8.18. It is also called the **local sidereal time** (LST). Hence,

$$HA\Upsilon = \text{LST}.$$

If X is the position of a star, its meridian PX meeting the equator at the point B , then we have:

$$\begin{aligned} \text{right ascension of } X &= \text{arc } \Upsilon B \\ \text{hour angle of } X &= \angle ZPX = \text{arc } AB. \end{aligned}$$

But

$$A\Upsilon = \Upsilon B + BA$$

hence,

$$\text{hour angle of } X + \text{right ascension of } X = \text{local sidereal time}$$

or

$$\mathbf{HAX + RAX = LST.} \tag{8.7}$$

This is an important relationship for X can be any celestial object—star, Sun, Moon, planet, even an artificial satellite or spacecraft. If any two of the three quantities in equation (8.7) are known, the third can be calculated. We will consider the implications of this later when we look at sidereal time in more detail.

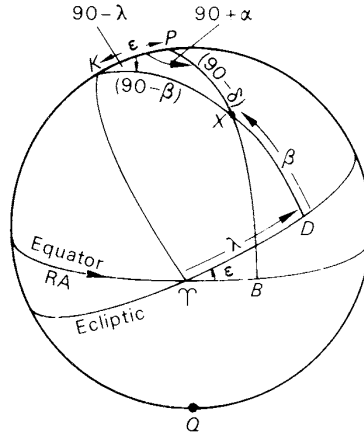


Figure 8.19. Ecliptic coordinates.

8.14 The ecliptic system of coordinates

This system is specially convenient in studying the movements of the planets and in describing the Solar System. The two quantities specifying the position of an object on the celestial sphere in this system are ecliptic longitude and ecliptic latitude. In figure 8.19 a great circle arc through the pole of the ecliptic K and the celestial object X meets the ecliptic in the point D . Then the **ecliptic longitude**, λ , is the angle between Υ and D , measured from 0° to 360° along the ecliptic in the eastwards direction, that is in the direction in which right ascension increases. The **ecliptic latitude**, β , is measured from D to X along the great circle arc DX , being measured from 0° to 90° north or south of the ecliptic. It should be noted that the north pole of the ecliptic, K , lies in the hemisphere containing the north celestial pole. It should also be noted that ecliptic latitude and longitude are often referred to as **celestial latitude** and **longitude**.

The point of intersection Aries (Υ) of the celestial equator and the ecliptic is often referred to as the **ascending node**, since an object travelling in the plane of the ecliptic with the direction of increasing right ascension (eastwards) passes through Aries from southern to northern declinations. By similar reasoning, Libra (♎) is called the **descending node**.

The origins most often used with this system of coordinates are the Earth's centre and the Sun's centre since most of the planets move in planes inclined only a few degrees to the ecliptic.

It is often required to convert from the ecliptic system to equatorial coordinates, i.e. the system of right ascension and declination or *vice versa*. This may be achieved by considering the spherical triangle KPX in figure 8.19, where $\angle KPX = 90^\circ + \alpha$, α being the right ascension of X , or ΥB , while BX is the object's declination, δ .

Let us suppose α , δ are known, also the obliquity of the ecliptic, and it is required to calculate, λ , β . Then, using the cosine formula,

$$\cos(90 - \beta) = \cos \epsilon \cos(90 - \delta) + \sin \epsilon \sin(90 - \delta) \cos(90 + \alpha)$$

or

$$\sin \beta = \cos \epsilon \sin \delta - \sin \epsilon \cos \delta \sin \alpha. \tag{8.8}$$

Applying the cosine formula once more, we have

$$\cos(90 - \delta) = \cos \epsilon \cos(90 - \beta) + \sin \epsilon \sin(90 - \beta) \cos(90 - \lambda)$$

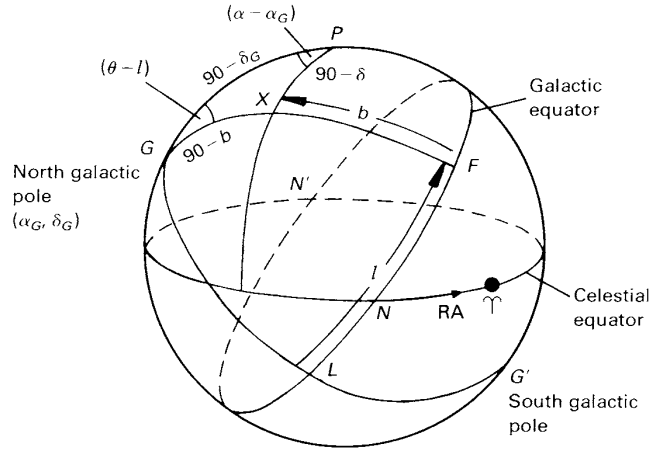


Figure 8.20. Galactic coordinates.

that is

$$\sin \delta = \cos \varepsilon \sin \beta + \sin \varepsilon \cos \beta \sin \lambda$$

or

$$\sin \lambda = \frac{\sin \delta - \cos \varepsilon \sin \beta}{\sin \varepsilon \cos \beta}. \quad (8.9)$$

Values for λ may be obtained directly from α, δ by substituting for β in equation (8.9), so providing a formula for the calculation of λ given by

$$\tan \lambda = \frac{\sin \alpha \cos \varepsilon + \tan \delta \sin \varepsilon}{\cos \alpha}. \quad (8.10)$$

The quadrant associated with λ can be elucidated by noting the signs of the numerator and denominator in either of the equations (8.9) or (8.10).

Alternatively, these identities could have been derived using the the four-parts formula. The solution of the problem in reverse (given $\lambda, \beta, \varepsilon$; find α, δ) is left to the reader.

8.15 Galactic coordinates

In the same way that it is convenient to use ecliptic coordinates in problems dealing with motions of planets about the Sun, it is often convenient in studies of the distribution and movements of the bodies in the stellar system to which the Sun belongs, to use a coordinate system based on the observational fact that the Galaxy is lens-shaped with the Sun in or near to the median plane of the lens.

The fact that the Milky Way is a band of light occupying a great circle supports this view.

We can take the Galaxy as being symmetrically distributed on either side of the galactic equator $LN'F$ (figure 8.20) which intersects the celestial equator in the two points N and N' . These points are referred to as the **ascending** and **descending nodes** respectively, since an object travelling in the galactic equator and passing through N in the direction of increasing RA, ascends from the southern to the northern hemisphere. In passing through N' , it descends from the northern to southern hemisphere. Similarly, the north galactic pole G is the galactic pole lying in the northern hemisphere.

Any object X ($RA = \alpha, Dec = \delta$) has galactic coordinates in latitude and longitude.

Table 8.1. Position of the Galactic Pole.

IAU galactic pole (N) ($b^I = +90^\circ$)	
$\alpha = 12^{\text{h}} 51^{\text{m}} 4$	$\delta = +27^\circ 07'.7$ (Epoch 2000)
$\alpha = 12^{\text{h}} 49^{\text{m}} 0$	$\delta = +27^\circ 24'.0$ (Epoch 1950)
$\alpha = 12^{\text{h}} 46^{\text{m}} 6$	$\delta = +27^\circ 40'.0$ (Epoch 1900)
Ohlsson galactic pole (N) ($b^I = +90^\circ$)	
$\alpha = 12^{\text{h}} 40^{\text{m}}$	$\delta = +28^\circ 0$ (Epoch 1900)

Galactic longitude, l , is measured along the galactic equator to the foot of the meridian from G through the object from 0° to 360° in the direction of increasing right ascension. Prior to 1959, the zero was the ascending node N (the *Ohlsson System*); since 1959, it is L , the point where the semi-great circle from G at the position angle $\theta = PGL = 123^\circ (= 90^\circ + 33^\circ)$ meets the galactic equator. This seemingly arbitrary angle is taken so that L lies in the direction of the galactic centre. Thus, the galactic longitude l of X (figure 8.20) is $\angle LNF$ and $\angle PGX$ is $(\theta - l)$.

Galactic latitude, b , is measured north and south of the galactic equator from 0° to 90° along with the semi-great circle from the north galactic pole through the object to the equator. Thus, the galactic latitude b of X is arc FX and is north.

To differentiate between the old (Ohlsson) and newer (IAU) systems of galactic coordinates, it is usual to label l and b with superscripts I and II respectively. Table 8.1 summarizes the position of the north galactic pole in the two systems, the changes with time of the IAU system being caused by precession of the equinoxes (see section 11.9).

A typical conversion problem is to find the galactic longitude l and galactic latitude b of an object X of known right ascension α and declination δ , given the coordinates of the north galactic pole $G(\alpha_G, \delta_G)$ and the position angle θ .

Spherical triangle PGX shows that as before the cosine formula, applied twice, can obtain the desired quantities. Thus,

$$\cos(90 - b) = \cos(90 - \delta_G) \cos(90 - \delta) + \sin(90 - \delta_G) \sin(90 - \delta) \cos(\alpha - \alpha_G)$$

or

$$\sin b = \sin \delta_G \sin \delta + \cos \delta_G \cos \delta \cos(\alpha - \alpha_G) \quad (8.11)$$

giving b . Also,

$$\cos(90 - \delta) = \cos(90 - \delta_G) \cos(90 - b) + \sin(90 - \delta_G) \sin(90 - b) \cos(\theta - l)$$

that is

$$\sin \delta = \sin \delta_G \sin b + \cos \delta_G \cos b \cos(\theta - l)$$

or, rearranging,

$$\cos(\theta - l) = \frac{\sin \delta - \sin \delta_G \sin b}{\cos \delta_G \cos b}. \quad (8.12)$$

Again, as in the case of the conversion of α, δ to ecliptic coordinates, values for $\sin b$ and $\cos b$ taken from equation (8.11) may be substituted into equation (8.12) leading to the identity

$$\tan(\theta - l) = \frac{\tan \delta \cos \delta_G - \cos(\alpha - \alpha_G) \sin \delta_G}{\sin(\alpha - \alpha_G)}. \quad (8.13)$$

Knowing the value of θ , we can calculate l .

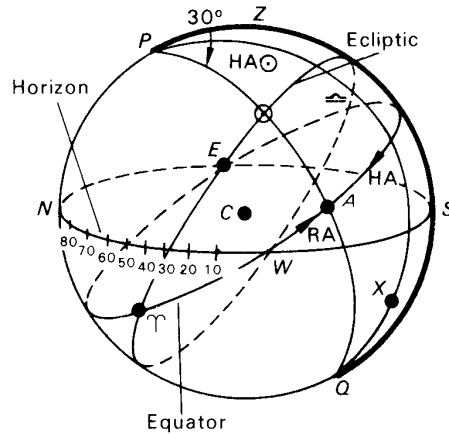


Figure 8.21. Example 8.4—the celestial sphere for an observer in latitude 60° N at LST 9^{h} on June 21st.

We now consider some celestial sphere problems embodying the concepts of the last few sections in this chapter. Since the problems require estimations only, no calculation by spherical trigonometrical formulas is required. But by drawing a fair-sized celestial sphere (at least 100 mm in diameter), by remembering that foreshortening occurs in such drawings (see the 10° marks on arc NW in figure 8.21), and allowing for it in estimating angles and arcs, by remembering that all great circles are foreshortened into ellipses containing the centre of the sphere, values within a few degrees of the correct answers may easily be obtained. The very useful convention of using dotted lines for arcs on the back of the sphere should also be adhered to and the use of coloured pencils to distinguish one great circle from another is advised.

Example 8.4. Draw the celestial sphere for an observer in latitude 60° N at local sidereal time 9^{h} on June 21st. Put in the horizon, the equator, the four cardinal points, the observer's meridian, the ecliptic, the Sun and the position of a star X (right ascension 8^{h} , declination 50° S). What is the estimated hour angle of the Sun?

Draw the sphere. Insert the zenith, Z , and the horizon. The latitude $\phi = 60^\circ$ N, hence $90 - \phi = 30^\circ$ and P can be placed so that $PZ = 30^\circ$. Hence, Q and the equator may be put in. Thicken PZQ to indicate the observer's meridian. N is the point of intersection with the horizon of the vertical from Z through P . Now S , W and E can be put in the diagram, remembering that when facing north, W is on the observer's left-hand side. Put arrowheads in with RA and HA beside them on the equator to indicate the directions in which right ascension and hour angle are measured.

Insert Υ , using the fact that

$$HA\Upsilon = LST = 9^{\text{h}}.$$

Insert ♊ , the other point of intersection of the ecliptic with the equator. It must be on the other side of the sphere through the centre C .

Remember

- (i) that the obliquity of the ecliptic is $\sim 23\frac{1}{2}^\circ$ and
- (ii) that the Sun passes through Υ from south declination to north declination in the direction of increasing right ascension.

Hence, insert the ecliptic.

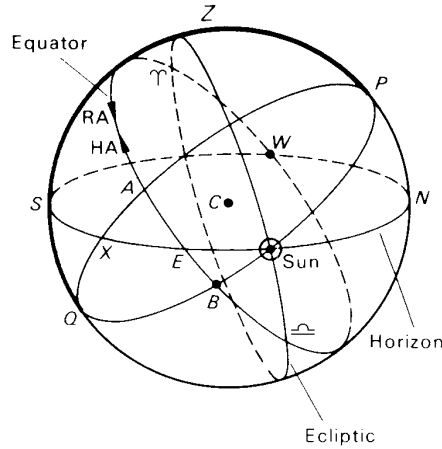


Figure 8.22. Example 8.5—the celestial sphere for latitude 30° N at the time of rising of *Sirius*.

Insert the Sun, ☉, knowing that on June 21st the Sun’s right ascension is about 6^h, that is $\Upsilon A \approx 90^\circ$.

Insert the star *X* according to the information given.

From figure 8.21, the hour angle of the Sun (HA_{\odot}) is estimated to be about 3^h.

In fact, since $RA_{\odot} = 6^h$ and $LST = 9^h$, we could have obtained the HA_{\odot} from the relation

$$LST = HA_{\odot} + RA_{\odot}$$

or

$$HA_{\odot} = LST - RA_{\odot} = 9^h - 6^h = 3^h.$$

Example 8.5. Draw the celestial sphere for latitude 30° N, showing the star *Sirius* (right ascension 6^h 40^m, declination 17° S) at rising and draw the ecliptic. Estimate from your diagram the approximate date when *Sirius* rises with the Sun.

Again a sphere is drawn (see figure 8.22), the zenith and horizon are inserted and we note that ZP this time is 60°. Because we are considering the rising of a star and of the Sun, it is more convenient to have the east point *E* on the front of the diagram. This dictates the position of the north point *N* so that the west point is on the observer’s left-hand side when facing north. *W* and *S* are now inserted. The north celestial pole can now be put in between *N* and *Z* and such that $NP = 30^\circ$. *Q* and the equator are drawn in, the observer’s meridian PZQ is indicated and arrowheads with *RA* and *HA* are added to the equator, their directions being fixed by remembering that hour angle is always measured from the observer’s meridian westwards.

Sirius is rising, therefore it must be on the horizon: it has declination 17° S so it must lie on a parallel of declination set 17° from the equator in the southern hemisphere. Hence, *Sirius* is at *X*, where $AX = 17^\circ$.

We now insert the ecliptic. Υ is found by noting that it must be on the equator at a point such that the right ascension of *X*, ΥA , is $6\frac{2}{3}^h$. Having fixed Υ to accord with this, \sphericalangle is then put in. Using the convention illustrated by figure 8.23 and remembering that the value of the obliquity is about $23\frac{1}{2}^\circ$, we draw in the ecliptic.

The Sun must be at ☉ since it is rising and always lies on the ecliptic.

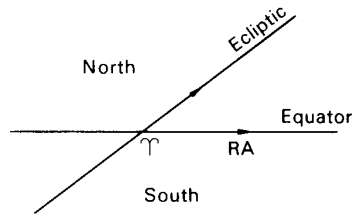


Figure 8.23. Example 8.5—the convention of the ascending node of the ecliptic (Υ) as viewed from the outside of the celestial sphere.

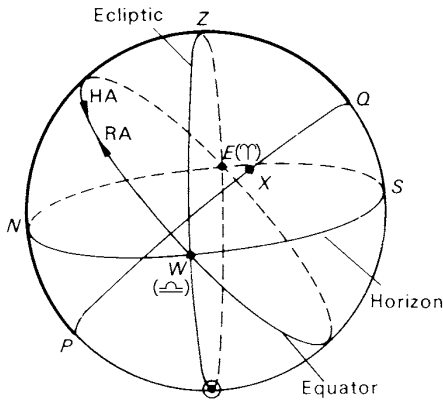


Figure 8.24. Example 8.6—the celestial sphere for latitude 30° S showing the ecliptic.

Its right ascension, ΥB , is, therefore, about $8\frac{2}{3}^h$. But its right ascension increases by 24^h in 12 months or 2^h per month.

The Sun's right ascension being 6^h about June 21st, it will have increased a further $2\frac{2}{3}$ hours in 1 month 10 days after June 21st, that is about August 1st.

Hence, the approximate date when *Sirius* rises with the Sun for the given latitude is August 1st.

Example 8.6. Draw the celestial sphere for an observer in latitude 30° S, showing the Sun, the ecliptic and the First Point of Aries at apparent midnight on June 21st. Estimate the local sidereal time. Show also the position of a star of right ascension 13^h and declination 30° S.

First we draw the sphere and insert zenith Z and the horizon. Inserting the four cardinal points N, E, S, W , we place the south celestial pole Q at an altitude of 30° above the south point since, by definition, the great circle from Z through Q cuts the horizon in the south point.

We place the north celestial pole P in the diagram, insert the equator and the observer's meridian QZP . Arrowheads are put in with HA and RA beside them to indicate the directions in which hour angle and right ascension are measured, remembering that hour angle is always measured from the observer's meridian westwards.

It is June 21st so that the Sun's declination is $23^\circ 26' N$; it is apparent midnight, i.e. $HA_\odot = 12^h$. Hence, in figure 8.24, the Sun's position is given by \odot .

The Sun's right ascension on June 21st is $6^h \equiv 90^\circ$. Hence, the ecliptic must pass through \odot and intersect the equator 90° before and behind \odot in right ascension. The only possible points are W and

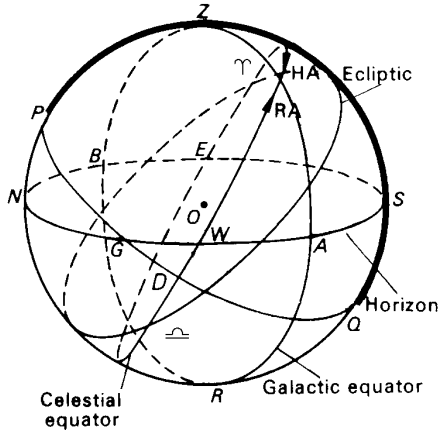


Figure 8.25. Example 8.7—the celestial for latitude 28° N displaying galactic coordinates.

E. Draw in the ecliptic. By making use of the definition embodied in figure 8.23, we can see that E is Υ and W is $\underline{\cap}$.

$$\text{The local sidereal time} = \text{HA}\Upsilon = 18^{\text{h}}.$$

The star's position X is then inserted according to the information given.

Example 8.7. Draw the celestial sphere for latitude 28° N, and insert the north galactic pole G (declination 28° N) when it is setting. Insert the galactic equator at this instant. Estimate the hour angle of G at setting and state the local sidereal time, the right ascension of G being $12^{\text{h}} 47^{\text{m}}$. Insert the First Point of Aries in the diagram and draw the ecliptic.

Show that, once in each sidereal day, the galactic equator and the horizon coincide in this latitude. At what sidereal time does this occur? At what approximate date would this occur at midnight?

As usual we put the zenith Z and the horizon into the sphere. The problem deals with setting of a celestial object, therefore we wish the west point in W on the front of the diagram (figure 8.25). This dictates the positioning of the other cardinal points N , E and S . The altitude of the pole being the latitude of the observer, P is placed 28° above N and then the horizon, the south celestial pole Q , the observer's meridian and the arrowheads for HA and RA can be inserted.

The galactic pole G is setting, so is placed on the horizon such that GD is 28° N. In drawing the galactic equator at this moment we remember that G is its pole, i.e. all points on the galactic equator are 90° from G . We choose A and B to be two such points; Z and the nadir R must also be 90° from G for they are the poles of the great circle on which G lies, namely the horizon. Hence, the galactic equator can now be drawn in, passing through the zenith, Z .

$$\text{The hour angle of } G = \angle ZPD \approx 7^{\text{h}}.$$

$$\text{LST} = \text{RAG} + \text{HAG} = 12^{\text{h}} 47^{\text{m}} + 7^{\text{h}} = 19^{\text{h}} 47^{\text{m}} \text{ approximately.}$$

Υ is placed so that the arc $\Upsilon ED = 12^{\text{h}} 47^{\text{m}}$; Libra ($\underline{\cap}$) is then inserted and, using figure 8.23 and the obliquity value of $23\frac{1}{2}^{\circ}$, the ecliptic can be put in.

Since G in the course of a **sidereal day** (the time between two successive passages of Υ across the observer's meridian) follows the parallel of declination 28° N, and $PZ = 90 - \phi = 62^{\circ}$, Z lies on his parallel. Hence, once per sidereal day, G coincides with Z , i.e. the pole of the galactic equator

coincides with the pole of the horizon. By definition, then, the galactic equator and horizon must coincide at this time.

The hour angle of G at which this occurs is zero. But the RAG is $12^{\text{h}} 47^{\text{m}}$. Hence,

$$\text{LST} = \text{RAG} + \text{HAG} = 12^{\text{h}} 47^{\text{m}} + 0^{\text{h}} = 12^{\text{h}} 47^{\text{m}}.$$

To occur at midnight, the hour angle of the Sun would be 12^{h} . But

$$\begin{aligned} \text{RA}_{\odot} &= \text{LST} - \text{HA}_{\odot} \\ &= 12^{\text{h}} 47^{\text{m}} - 12^{\text{h}} = 47^{\text{m}}. \end{aligned}$$

The Sun's right ascension increases by 24^{h} in 12 months or 2^{h} per month. It, therefore, increases from 0^{h} (March 21st) to $47^{\text{m}} \sim \frac{3}{4}^{\text{h}}$ in approximately 11^{d} . The date is, therefore, around April 1st.

Problems—Chapter 8

Note: In the following problems, assume (i) a spherical Earth, (ii) the obliquity of the ecliptic to be $23^{\circ} 26'$. Although problems 5 to 16 require the use of spherical trigonometry, the student without trigonometry will find it instructive to try to solve them approximately by drawing the relevant celestial spheres. The student's estimates can be compared with the answers given in the Answer Appendix.

1. Draw the celestial sphere for an observer in latitude 45° S, putting in the observer's meridian, the four cardinal points and a star of azimuth 300° E of S and altitude 30° . *Estimate* the star's right ascension and declination if the local sidereal time at that instant is 9^{h} . Insert the ecliptic. If it is also apparent midnight, *estimate* the date. On what date (approximately) will the star set when the Sun sets?
2. Draw the celestial sphere for an observer in latitude 55° S, showing the positions of two stars X (altitude 40° , azimuth 130° E of S) and Y ($\text{HA} = 19^{\text{h}}$, $\text{Dec} = 40^{\circ}$ S). *Estimate* from your diagram the hour angle and declination of X and the altitude and azimuth of Y .
If the local sidereal time is 10^{h} , sketch the ecliptic in your diagram; *estimate* the celestial longitudes and latitudes of the two stars and *estimate* the approximate date on the assumption that the Sun is rising at this moment.
3. Draw the celestial sphere for an observer in latitude 30° N, putting in the horizon, equator, zenith, north and south celestial poles and the observer's meridian.
Show the positions of two stars X and Y as follows:

X : hour angle 3^{h} ; declination 64° N

Y : azimuth 120° W; altitude 20° .

From the diagram *estimate* (i) the azimuth and altitude of X , (ii) the hour angle and declination of Y . If the right ascension of X is 6^{h} , insert the ecliptic.

A traveller states that when he/she was in that latitude the Sun passed through his/her zenith. Give a reason for believing or disbelieving him/her. Show that star X is a circumpolar star.

4. Draw the celestial sphere for an observer in latitude $23^{\circ} 26'$ N, inserting the horizon, equator, zenith, north and south celestial poles and the observer's meridian. Insert the First Point of Aries and the ecliptic when the Sun is rising on June 21st. Put in the Sun's position when it transits on that day (i.e. at apparent noon). *Estimate* the local sidereal time at apparent noon, also the altitude and azimuth at that time of a star X whose right ascension and declination are 10^{h} and 65° N respectively. Is the star circumpolar?
5. Show that the celestial longitude λ and latitude β of a star can be expressed in terms of its right ascension α and declination δ by the formulas

$$\begin{aligned} \sin \beta &= \sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \sin \alpha \\ \cos \beta \cos \lambda &= \cos \delta \cos \alpha \\ \cos \beta \sin \lambda &= \sin \lambda \sin \varepsilon + \cos \delta \cos \varepsilon \sin \alpha \end{aligned}$$