CONFERENCIA

Dragging the roots of a polynomial to the unit circle

Arnaldo Mandel
Profesor Universidad de Sao Paulo

Fecha: Miércoles 28 de agosto de 2019
Hora: 2:00p.m.
Lugar: Salón 201, edificio Yu Takeuchi (404)

Departamento de Matemáticas
Facultad de Ciencias
Sede Bogotá

UNIVERSIDAD NACIONAL DE COLOMBIA

Abstract

Several conditions are known for a self-inversive polynomial that ascertain the location of its roots, and we present a framework for comparison of those conditions. We associate a parametric family of polynomials p_α to each such polynomial p, and define $cn(p)$, $il(p)$ to be the sharp threshold values of α that guarantee that, for all larger values of the parameter, p_α has, respectively, all roots in the unit circle and all roots interlacing the roots of unity of the same degree. Interlacing implies circle rootedness, hence $il(p) \geq cn(p)$, and this inequality is often used for showing circle rootedness. Both $cn(p)$ and $il(p)$ turn out to be semi-algebraic functions of the coefficients of p, and some useful bounds are also presented, entailing several known results about roots in the circle. The study of $il(p)$ leads to a rich classification of real self-inversive polynomials of each degree, organizing them into a complete polyhedral fan. We have a close look at the class of polynomials for which $il(p)=cn(p)$, whereas in general the quotient $il(p)$ $cn(p)$ is shown to be unbounded as the degree grows.

Several examples and open questions are presented.

Departamento de Matemáticas
Facultad de Ciencias
Sede Bogotá

UNIVERSIDAD NACIONAL DE COLOMBIA